# Pauli Matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The Pauli matrices $\sigma_i$ are hermitian and unitary. Eigenvalues are -1,1.

$$\sigma_i \sigma_j = \delta_{ij} + \mathrm{i}\epsilon_{ijk}\sigma_k$$

$$\begin{gathered} \sigma_x \sigma_y = -\sigma_y \sigma_x = \mathrm{i}\sigma_z \\ \sigma_x^2 = \sigma_y^2 = \sigma_z^2 = \mathbb{1} \\ [\sigma_x, \sigma_y] = 2\mathrm{i}\sigma_z \\ \{\sigma_x, \sigma_y\} = 0 \end{gathered}$$

$$\begin{gathered} \mathrm{Tr}(\sigma_i) = 0 \\ \det(\sigma_i) = -1 \end{gathered}$$

$$\sigma_+ = \frac{1}{2}(\sigma_x + \mathrm{i}\sigma_y) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\sigma_- = \frac{1}{2}(\sigma_x - \mathrm{i}\sigma_y) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_x = \sigma_+ + \sigma_- \qquad \sigma_y = \frac{1}{\mathrm{i}}(\sigma_+ - \sigma_-)$$

$$\sigma_+\sigma_- = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad \sigma_-\sigma_+ = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathbb{1} = \sigma_+\sigma_- + \sigma_-\sigma_+ \qquad \sigma_z = \sigma_+\sigma_- - \sigma_-\sigma_+ \quad 2\sigma_+\sigma_- = \mathbb{1} + \sigma_z$$